Solution of Lane-Emden Type Equations Using Polynomial-Sinc Collocation Method

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Abstract

The Lane-Emden (LE) equation is one of the basic equations in the theory of stellar structure and has been the focus of several studies [1,2,3]. In general, LE type equations are nonlinear ODEs that can be formulated as:

$$y'' + \frac{2}{x}y' + g(y) = R(x),\ x > 0 \tag{1}$$

with the initial conditions,

$$y(0) = a,\ y'(0) = b.$$

In general the generic function $g(y)$ is nonlinear and $R(x)$ is a function of x only. Many problems in mathematical physics and astrophysics are related to this equation. Some of these applications are homogeneous, $R(x) = 0$, and others are inhomogeneous. For every physical application a suitable choice of the generic function $g(y)$ is made.

One of the main problems with the LE equation is the singularity at $x = 0$ which is a singularity at the boundary as well as of the equation. This singularity was a challenge for many scholars to numerically represent the solution of the LE equation.

In this paper we introduce a series solution based on Lagrange polynomials that can easily deal with singularity problems. The proposed approximation is based on non-equidistant interpolation points generated by conformal maps. Our method provides the solution by an exponential convergent series. This exponential convergence property arises from the use of Sinc points as interpolation points in the Lagrange polynomials. We examine the technique for different types of Emden' equations and compare the solution with Taylor approximation. In addition the error formula shows exponential convergence.

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