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New centroid index for ordering fuzzy numbers

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Abstract— Ranking fuzzy numbers is an important tool in decision process. In fuzzy decision analysis, fuzzy quantities are used to describe the performance of alternative in modeling a real-world problem. Most of the ranking procedures proposed so far in literature cannot discriminate fuzzy quantities and some are counterintuitive. As fuzzy numbers are represented by possibility distributions, they may overlap with each other, and hence it is not possible to order them. It is true that fuzzy numbers are frequently partial order and cannot be compared like real numbers which can be linearly ordered. So far, more than 100 fuzzy ranking indices have been proposed since 1976 while the theory of fuzzy sets was first introduced by Zadeh. The most commonly used approached for ranking fuzzy numbers is ranking indices based on centroid of fuzzy numbers. Ever since Yager presented the centroid concept in the ranking techniques using the centroid concept have been proposed and investigated. In a paper by Cheng, a centroid-based distance method presented. The method utilized the Euclidean distances from the origin to the centroid point of each fuzzy numbers to compare and rank the fuzzy numbers. Chu and Tsao found that the distance method could not rank fuzzy numbers correctly if they are negative and therefore, suggested using the area between centroid point and the origin to rank fuzzy numbers. Deng et al. utilized the centroid point of a fuzzy number and presented a new area method to rank fuzzy numbers with the radius of gyration (ROG) points to overcome the drawback of the Cheng's distance method and Tsao's area method when some fuzzy numbers have the same centroid point. However, ROG method cannot rank negative fuzzy numbers. Recently, Wang et al. pointed out that the centroid point formulas for fuzzy numbers provided by Cheng are incorrect and have led to some misapplication such as by Chu and Tsao, Pan and Yeh and Deng et al.. They presented the correct centroid formulae for fuzzy numbers and justified them from the viewpoint of analytical geometry. Nevertheless, the main problem, about ranking fuzzy numbers methods, which used the centroid point, was reminded. In 2008, Wang and Lee revised Chu and Tsao's method and suggested a new approach for ranking fuzzy numbers based on Chu and Tsao's method in away to similar original point. However, there is a shortcoming in some situations. In 2011, Abbasbandy and Hajjari improved cheng's distance method. Afterward, Pani Bushan Rao et al. presented a new method for ranking fuzzy numbers based on the circumcenter of centroids and used an index of optimism to reflect the decision maker's optimistic attitude and also an index of modality that represented the neutrality of the decision maker. However, there are some weaknesses associated with these indices. This paper proposes a new centroid index ranking method that is capable of effectively ranking various types of fuzzy numbers. The contents herein present several comparative examples demonstrating the usage and advantages of the proposed centroid index ranking method for fuzzy numbers.

***Index Terms*— Centroid points, Comparison, Decision-making, Defuzzification, Fuzzy numbers, Ordering;**

I. PRELIMINARIES

In this section, we briefly review some basic concepts of generalized fuzzy numbers and some existing methods for ranking fuzzy numbers. We will identify the name of the number with that of its membership function for simplicity. Throughout this paper, R stands for the set of all real numbers, E stands the set of fuzzy numbers, "A" expresses a fuzzy number and $A(x)$ for its membership function, $\forall x \in R$.

A. Basic notations and definitions

A generalized fuzzy number "A" is a subset of the real line R, with membership function $A(x): R \to [0,\omega]$ such that [22]:

$$\mu_A(x) = \begin{cases} L_A(x) & a \le x \le b \\ \omega & b \le x \le c \\ U_A(x) & c \le x \le d \\ 0 & otherwise, \end{cases} \tag{1}$$

Where $0 \le \omega \le 1$ is a constant, and $L_A:[a,b] \to [0,\omega]$ and $U_A:[c,d] \to [0,\omega]$ are two strictly monotonically and continuous mapping from R to closed interval $[0,\omega]$. If $\omega = 1$, then A is a normal fuzzy number; otherwise, it is a trapezoidal fuzzy number and is usually denoted by $A = (a,b,c,d,\omega)$ or $A = (a,b,c,d)$ if $\omega = 1$.

In particular, when $b = c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by $A = (a,b,d,\omega)$ or $A = (a,b,d)$ if $\omega = 1$.

Therefore, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers. We show the set of generalized fuzzy numbers by $F_{\omega}(R)$ or for simplicity by $F(R)$.

Since $L_A(x)$ and $U_A(x)$ are both strictly monotonically and continuous functions, their inverse functions exist and should be continuous and strictly monotonical. Let $A_L:[0,\omega]\to[a,b]$ and $A_U:[0,\omega]\to[c,d]$ be the inverse functions of $L_A(x)$ and $U_A(x)$, respectively. Then A_L and A_U should be integrable on the close interval $[0,\omega]$. In other words, both $\int_0^{\omega} A_L(y)dy$ and $\int_0^{\omega} A_U(y)dy$ should exist. In the case of trapezoidal fuzzy number, the inverse functions A_L and A_U can be analytically expressed as

$$A_L(y) = a + (b-a)y/\omega, \quad 0 \le y \le \omega \tag{2}$$

$$A_U(y) = d - (d-c)y/\omega, \quad 0 \le y \le \omega \tag{3}$$

The functions $L_A(x)$ and $U_A(x)$ are also called the left and right side of the fuzzy number A respectively [22].

In this paper, we assume that

$$\int_{-\infty}^{+\infty} A(x)dx < +\infty.$$

A useful tool for dealing with fuzzy numbers are their α-cuts. The α-cut of a fuzzy number A is non-fuzzy set defined as

$$A_{\alpha} = \{x \in R : A(x) \ge \alpha\},$$

for $\alpha \in (0.1]$ and $A_0 = cl\left(\bigcup_{\alpha\in(0,1]} A_{\alpha}\right)$. According to the definition of a fuzzy number, it is seen at once that every α-cut of a fuzzy number is closed interval. Hence, for a fuzzy number A, we have

$$A(\alpha) = [A_L(\alpha), A_U(\alpha)]$$

where

$$A_L(\alpha) = \inf\{x \in R : A(x) \ge \alpha\},$$

$$A_U(\alpha) = \sup\{x \in R : A(x) \ge \alpha\}.$$

If the left and right sides of the fuzzy number A are strictly monotone, as it is described, A_L and A_U are inverse functions of $L_A(x)$ and $U_A(x)$, respectively.

The set of all elements that have a nonzero degree of membership in A, it is called the support of A, i.e.

$$\text{Supp}\,(A) = \{x \in X \mid A(x) \succ 0\} \tag{4}$$

The set of elements having the largest degree of membership in A, it is called the core of A, i.e.

$$\text{Core}\,(A) = \left\{x \in X \mid A(x) = \sup_{x\in X} A(x)\right\} \tag{5}$$

In the following, we will always assume that A is continuous and bounded support Supp(A). The strong support of A should be Supp$(A) = [a,d]$.

Definition 1.1 A function $s:[0,1]\to[0,1]$ is a reducing function if is s increasing and $s(0)=0$ and $s(1)=1$. We say that s is a regular function if $\int_0^1 s(r)dr = \frac{1}{2}$.

Definition 1.2 If A is a fuzzy number with r-cut representation, $[A_L(\alpha), A_U(\alpha)]$ and s is a reducing function, then the value of A (with respect to s); it is defined by

$$Val(A) = \int_0^1 s(\alpha)[A_U(\alpha) + A_L(\alpha)]d\alpha. \tag{6}$$

Definition 1.3 If A is a fuzzy number with r-cut representation $[A_L(\alpha), A_U(\alpha)]$, and s is a reducing function then the ambiguity of A (with respect to s) is defined by

$$Amb(A) = \int_0^1 s(\alpha)[A_U(\alpha) + A_L(\alpha)]d\alpha. \tag{7}$$

B. Arithmetic operation

In this subsection, arithmetic operation between two generalized trapezoidal fuzzy numbers, defined on universal set of real numbers R, are reviewed [23].

Let $A = (a_1, b_1, c_1, d_1; \omega_1)$ and $A = (a_2, b_2, c_2, d_2; \omega_2)$ be two generalized trapezoidal fuzzy numbers then

$$A_1 \oplus A_2 = \begin{pmatrix} a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \\ \min(\omega_1, \omega_2) \end{pmatrix}$$

$$A_1 - A_2 = \begin{pmatrix} a_1 - d_2, b_1 - c_2, c_1 - b_2, \\ d_1 - a_2; \\ \min(\omega_1, \omega_2) \end{pmatrix}$$

$$\lambda A_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; \omega_1) & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; \omega_1) & \lambda < 0 \end{cases}$$

II. Review on some Centroid-index for ordering fuzzy numbers

[21] was the first researcher to proposed a centroid-index ranking method to calculate the value x_0 for a fuzzy number A as

$$x_0 = \frac{\int_0^1 \omega(x) f_A(x)dx}{\int_0^1 f_A(x)dx} \tag{8}$$

where $\omega(x)$ is a weighting function measuring the importance of the value x and f_A denotes the membership function of the fuzzy number A.

When $\omega(x) = x$, the value x_0 becomes the geometric Center of Gravity (COG) with

$$x_0 = \frac{\int_0^1 x f_A(x)dx}{\int_0^1 f_A(x)dx} \tag{9}$$

The larger the value is of x_0 the better ranking of A.

Cheng [6] used a centroid-based distance approach to rank fuzzy numbers. For trapezoidal fuzzy number $A = (a,b,c,d;\omega)$, the distance index can be defined as

$$R(A) = \sqrt{{x_0}^2 + {y_0}^2}, \tag{10}$$

where

$$x_0 = \frac{\int_a^b xL_A(x)dx + \int_b^c xdx + \int_c^d xU_A(x)dx}{\int_a^b L_A(x)dx + \int_b^c dx + \int_c^d U_A(x)dx}, \tag{11}$$

$$y_0 = \omega \frac{\int_0^1 yA_L(y)dx + \int_0^1 yA_U(y)dy}{\int_0^1 A_L(y)dx + \int_0^1 A_U(y)dy}. \tag{12}$$

U_A and L_A are the respective right and left membership function of A, and A_U and A_L, are the inverse of U_A and L_A respectively. The larger the value is of $R(A)$ the better ranking will be of A.

Chau and Tsao [7] found that the distance approach by Cheng [6] had shortcomings. Hence to overcome the problems, Cha and Tsao [7] proposed a new ranking index function $S(A) = x_0.y_0$, where x_0 is defined in Cheng [6] and

$$y_0 = \frac{\int_0^\omega yA_L(y)dx + \int_0^\omega yA_U(y)dy}{\int_0^\omega A_L(y)dx + \int_0^\omega A_U(y)dy}. \tag{13}$$

The larger the value is of $S(A)$ the better ranking will be of A.

In some special cases, Cha and Tsao's [7] approach also has the same shortcoming of Cheng's [6] and Cha and Tsao's centroid-index are as follows. For fuzzy numbers A, B, C and $-A, -B, -C$ according to Cheng's centroid-index $R(A) = \sqrt{{x_0}^2 + {y_0}^2}$, whereby the same results are obtained, that is, if $A < B < C$ then $-A < -B < -C$. This is clearly inconsistent with the mathematical logic. For Chu and Tesao's centroid-index $S(A) = x_0.y_0$, if $x_0 = 0$, then the value of $S(A) = x_0.y_0$, is a constant zero. In other words, the fuzzy numbers with centroid $(0, y_1)$ and $(0, y_2)$, $y_1 \neq y_2$ are considered the same. This is also obviously unreasonable.

In a study conducted by Wang et al. [24], the centroid formulae proposed by Cheng [6] is shown to be incorrect. Therefore to avoid many misapplication, Wang et al. [24] presented the correct centroid formulae as

$$x_0 = \frac{\int_a^b xL_A(x)dx + \int_b^c xdx + \int_c^d xU_A(x)dx}{\int_a^b L_A(x)dx + \int_b^c dx + \int_c^d U_A(x)dx}, \tag{14}$$

and

$$y_0 = \frac{\int_0^\omega yA_U(y)dx + \int_0^\omega yA_L(y)dy}{\int_0^\omega A_U(y)dx + \int_0^\omega A_L(y)dy}. \tag{15}$$

For an arbitrary trapezoidal fuzzy number $A = (a,b,c,d;\omega)$, the centroid point (x_0, y_0) is defined as [24]

$$x_0 = \frac{1}{3}\left[a + b + c + d - \frac{dc - ab}{(d+c) - (a+b)}\right] \tag{16}$$

$$y_0 = \frac{\omega}{3}\left[1 + \frac{c-d}{(d+c) - (a+b)}\right]. \tag{17}$$

In special case, when $b = c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number and formulas (18) and (19) will be simplified as follows, respectively.

$$x_0 = \frac{1}{3}\left[a + b + d\right] \tag{18}$$

$$y_0 = \frac{1}{3}. \tag{19}$$

III. New centroid-index methods

In this section the centroid point of a fuzzy number corresponds to a x_0 value on the horizontal axis and y_0 value on the vertical axis. The centroid point (x_0, y_0) for a fuzzy number A is as defined [24]:

$$x_0 = \frac{\int_a^b xL_A(x)dx + \int_b^c xdx + \int_c^d xU_A(x)dx}{\int_a^b L_A(x)dx + \int_b^c dx + \int_c^d U_A(x)dx} \tag{20}$$

$$y_0 = \frac{\int_0^\omega yA_U(y)dx + \int_0^\omega yA_L(y)dy}{\int_0^\omega A_U(y)dx + \int_0^\omega A_L(y)dy}. \tag{21}$$

For trapezoidal fuzzy number $A = (a,b,c,d;\omega)$, the centroid point (x_0, y_0) is defined as in \cite{YWJW}:

$$x_0 = \frac{1}{3}\left[a + b + c + d - \frac{dc - ab}{(d+c) - (a+b)}\right] \tag{22}$$

$$y_0 = \frac{\omega}{3}\left[1 + \frac{c-d}{(d+c) - (a+b)}\right]. \tag{23}$$

Since triangular fuzzy numbers are special cases of trapezoidal fuzzy number with $b = c$ for any triangular fuzzy numbers with a picewise linear membership function, its centroid can be determined by

$$x_0 = \frac{1}{3}\left[a + b + d\right] \tag{24}$$

$$y_0 = \frac{1}{3}\omega. \tag{25}$$

Definition 3.1 For generalized trapezoidal fuzzy number $A = (a,b,c,d;\omega)$ with the centroid point (x_0, y_0), the centroid-index associated with the ranking is defined as

$$I_{\alpha\beta} = \frac{\beta(x_0 + y_0)}{2} + (1-\beta)I_\alpha \tag{26}$$

where $\alpha, \beta \in [0,1]$. $I_{\alpha\beta}$ is the modality which represents the importance of ce ntral value against the extreme values x_0, y_0 and $I_{\alpha\beta}$. Here, β represent the weight of central value and $1-\beta$ is the weight associated with the extreme values x_0 and y_0. Moreover, $I_\alpha = \alpha y_0 + (1-\alpha)x_0$ is the index of optimism which represents the degree of optimism of a decision-makers. If $\alpha = 0$, we have a pessimistic decision maker's view point which is equal to the distance of the centroid point from Y-axis. If $\alpha = 1$, we have a optimistic decision maker's view point which is equal to the distance of the centroid point from X-axis, and when $\alpha = 0.5$, we have the moderate decision maker's view point and is equal to the mean of centroid point from Y and X axis. The larger value of α is, the higher the degree of the decision maker. The index of optimism is not alone sufficient to discriminate fuzzy numbers as this uses only extreme of the cicumcenterr of centroids. Hence, we upgrade this by using an index.

Definition 3.2 For generalized trapezoidal fuzzy number $A = (a,b,c,d;\omega)$ with the centroid point (x_0, y_0), the ranking function of the trapezoidal fuzzy number which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(A) = \sqrt{{x_0}^2 + {y_0}^2}$,, which is the Euclidean distance from the centroid point and original point. Using the above definitions we define ranking between fuzzy numbers as follows:

let A and B are two fuzzy numbers, then

(1) $R(A) > R(B)$ if and only if $A \succ B$,

(2) $R(A) < R(B)$ if and only if $A \prec B$,

(3) if $R(A) = R(B)$ then in this case the discrimination of fuzzy numbers is not possible.

Case (1) let A and B are two symmetric triangular fuzzy numbers with the same core, if $R(A) + 1/\delta_A > R(B) + 1/\delta_B$ then $A \succ B$ and if $R(A) + 1/\delta_A < R(B) + 1/\delta_B$ then $A \prec B$, where δ_A, δ_B are the spreads of A and B respectively.

Case (2) In such cases we use Definitions 4.1 and 4.2 to rank fuzzy numbers as Definition 4.2 alone is not sufficient to discriminate in all cases, that is, if $I_{\alpha\beta}(A) > I_{\alpha\beta}(B)$, then $A \succ B$, and if $I_{\alpha\beta}(A) < I_{\alpha\beta}(B)$, then $A \prec B$,

Remark 3.3 For two arbitrary trapezoidal fuzzy numbers A and B, we have

$$R(A+B) = R(A) + R(B).$$

Remark 3.4 For two symmetric triangular fuzzy numbers A and B, with the same core $A \succ B$, iff $\delta_A < \delta_B$.

Hence

$R(crisp``number) \le$

$R(triangular``simmetric``fuzzy``number)$.

For example, consider the crisp number $A = (0,0,0)$ and two symmetric fuzzy numbers $B = (-1,0,1), C = (-2,0,2)$. As we saw by applying Rezvani's approach the results will be $R(A) = R(B) = R(C) = 0.333$, and ranking order is $A = B = C$ However, as $\delta_A \to 0$ and $\delta_B = 0$, $\delta_C = 2$, ranking order is $R(A) > R(B) > R(C)$. In addition, to compare the crisp number $A = (1,1,1)$ and two symmetric fuzzy numbers $B = (0,1,2), C = (-1,1,3)$,we have the same result.

Remark 3.5 For all fuzzy numbers A, B, C and D we have

(1) $A \succ B$ then $A \oplus C \succ B \oplus C$

(2) $A \succ B$ then $A - B \succ B - C$

(3) $A \sim B$ then $A \oplus C \sim B \oplus C$

(4) $A \succ B, C \succ D$ then $A \oplus C \succ B \oplus D$

Example 3.6 The two fuzzy numbers $A = (0.1,0.2,0.4,0.5)$ and $B = (0.1,0.3,0.5)$ used in this example are adapted from Chen and Sanguansat's [25].

Fig. 1 shows the graphs of the two fuzzy numbers. The results obtained by the proposed approach and other approaches are shown in Table 1. It is worth mentioning that Yager's [26] approach, Cheng's [6] approach, Chu and Tsao's [7] approach, Chen and Sanguansat's [25] cannot differentiate A and B, that is, their ranking are always the same, i.e. $A \sim B$. Note that the ranking $A \prec B$ obtained by Murakami et al.'s [27] approach, Chen and Chen's [11] approach and Chen and Chen's [23] approach, are thought of as unreasonable and not consistent with human intuition due to the fact that the center of gravity of A is larger than the center of gravity of B on the Y-axis.

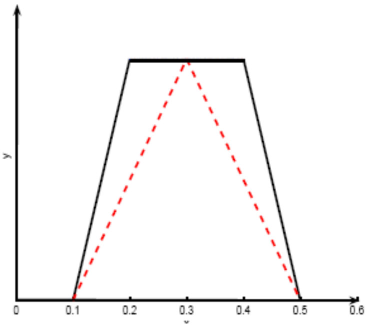

Fig. 1. Fuzzy number A and B in example 3.6.

Table 1 : Comparative results of Example 4.6

Ranking approach	A	B	Result
Yager [9]	0.3	0.3	$A \sim B$
Murakami et al. [11]	0.3	0.417	$A \prec B$
Cheng [25]	0.583	0.583	$A \sim B$
Chu and Tsao [29]	0.15	0.15	$A \sim B$
Chen and Chen [39]	0.424	0.446	$A \prec B$
Chen and Chen [44]	0.254	0.258	$A \prec B$
Chen and Sanguansat [45]	0.3	0.3	$A \sim B$
Phani Bushan Rao et al. [52]	0.4711	0.5026	$A \prec B$
Luu et al. [53]	0.3333	0.2222	$A \succ B$
Proposed method	0.6690	0.4484	$A \succ B$

IV. Conclusions

In spite of many ranking methods, no one can rank fuzzy numbers with human intuition consistently in all cases. Here, we pointed out the shortcoming of some recent centroid-index methods and presented a new centroid-index method for ranking fuzzy numbers.

The proposed formulae are simple and have cosistent expression on the horizontal axis and vertical axis and also be used for some especial cases in many centroid-index methods. The paper herein presents several comparative examples to illustrate the validity and advantages of proposed centroid-index ranking method. It shows that the ranking order obtained by the proposed centroid-index ranking method is more consistent with human intuitions than existing methods. Furthermore, the proposed ranking method can effectively rank a mix of varoius types of fuzzy numbers, which is another advantage of the proposed method over other existing ranking approaches.

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