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Calculation of the temperature distribution field in the deformation zone in metal rod rolling with the aid of locally homogeneous scheme

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Abstract - **On the basis of heat-conduction equation is solved the three-dimensional problem of temperature distribution field for continuous casting and rolling line of metal rod. Wherein considered the temperature rate change in rolling mill and intercellular gap of rolling mill line.**

In order to solve three dimensional problem of heat-conduction is used the score approximation method of locally one-dimensional schemes. With the aid of which multivariate problem reduced to locally one-dimensional problem. Calculation of temperature field distribution in deformation zone is executed under inherent scheme.

Key words: continuous casting, a metal rod, distribution field, deformation zone, stand, locally one-dimensional scheme.

I. Introduction

During metal forming under pressure one of the main factors, along with the speed, the deformation and stress that affect the quality of the products, is the temperature distribution field in the deformation zone. However, the experimental determination of the distribution of the temperature field in an industrial environment is almost impossible. Therefore, for temperature control in the manufacture of quality metal products the algorithm of solving three-dimensional problems of the temperature distribution field for the continuous casting line and metal rod rolling with the difference scheme was established.

Many scientists have been working with the problem of the temperature field model which describes temperature processes occurring in the deformation zone and technological process of the mill. In the creation of this algorithm the model [1] was used. In the practical calculations of the temperature in multistand mills a simplified model is used:

$$U_i = U_{i-1} - \Delta U_u - \Delta U_k - \Delta U_b + \Delta U_d, \quad (1)$$

if ΔU_u - radiation heat transfer to the environment, ΔU_k - convection heat transfer, ΔU_b - contact heat exchange with working rolls, ΔU_d - heating-up of metal by means of plastic energy.

In project [2], this model is implemented by using a finite element method.

During the numerical solution of temperature distribution field on the closed mill for a line of continuous casting and metal rod rolling, one of the most widely used method is the method of nets.

Simplicity and flexibility are characterized for the difference methods of solving boundary value problems of mathematical physics in the regular settlement areas, but the use of irregular nets brings together the finite difference method with the finite element method.

One of the most important advances in computational mathematics is the cost difference methods for solving multidimensional equations (with several space variables $x_1, x_2, \dots, x_n$) in partial derivatives [3]. One of the economical schemes class is additive schemes, which have total approximation, which is unconditionally stable and requires the cost of arithmetic operations Q, proportional to the grid nodes ω_h, in order to move from one layer to another.

The method of total approximation provides a completely stable converging locally one-dimensional scheme for parabolic equations.

Developed algorithm based on locally one-dimensional scheme, that implements a mathematical model (1), allows us to solve the problem of controlling the temperature conditions of rolling a metal rod on a rolling mill stand.

The problem is solved by a difference method, locally one-dimensional schemes [3] on the basis of the differential equation of heat conduction [2] in the form of:

$$c\rho \frac{\partial U}{\partial t} = div(\lambda \cdot gradU) + \tau_s H, \quad (2)$$

if c - thermal capacitance of metal; ρ - mass density of metal; λ - heat conduction coefficient; τ_s – metal resistance to plastic deformation of shift; H- intensity of shift deformation rate, with following boundary conditions:

1. At the entrance in the deformation zone the temperature of the metal is known;
2. On the free surface there is the heat transfer by radiation to the environment, which is described by the Stefan-Boltzmann formula;
3. On the contact surface there is the heat exchange between the surfaces of the deformable metal and the rolls, which is described by the expression:

$$\lambda \frac{\partial U}{\partial n} = \alpha_s (U - U_b), \quad (3)$$

if α_s- heat transfer coefficient on the contact surface; U_b - the temperature of rolls.

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4. On other surfaces, bounding ¼ of the deformation zone, the heat flow is zero.

In theory, in the most metal forming processes, the initial shape is different from the shape of the finished product, determined by the shape of the tool. The closer the element to the section angle, the less it receives the elongation. Therefore sectional sides receive convex shape. Square section will approach to the circular, and rectangular – firstly to the ellipsoid, and then to the circular. Since the draft gauges are designed for the gradual formation of the rolled profile, as well as rough gauges include simple shapes (rectangular, rhombus, oval, circle, square), the rectangular cross section was taken as the form of gauges of stands to experiment (Figure 1).

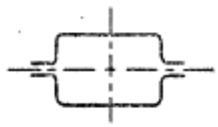


Fig. 1. Rectangular calibration rolls of roughing mill.

II. Difference equation

In order to construct the difference schemes for solving the heat conduction equation describing the temperature processes occurring in the deformation zone during the rolling of a metal rod, a three-dimensional parabolic equation of second order is consider [3]:

$$\frac{\partial U}{\partial t} = \sum_{\alpha=1}^{3} L_\alpha U + f(x,t),$$

$$L_\alpha U = \frac{\partial}{\partial x_\alpha}\left(k_\alpha(x,t)\frac{\partial U}{\partial x_\alpha}\right),$$

$$k_\alpha(x,t) \ge c_1 > 0, c_1 = const,$$

if $x = (x_1, x_2, x_3)$ - point of 3-dimensional space with x_1, x_2, x_3 coordinates. Let G – any 3-dimensional region with borders Γ, $\overline{G} = G + \Gamma$,

$$\overline{Q}_T = \overline{G}\times[0 \le t \le T], \quad Q_T = G\times(0 < t \le T].$$

The continuous solution of equation (6) in $\overline{Q}_T$ cylinder, satisfying the boundary condition

$$U = \mu(x,t) \text{ при } x \in \Gamma, 0 \le t \le T, \qquad (7)$$

and initial condition

$$U(x,0) = U_0(x), \text{ при } x \in \overline{G}. \qquad (8)$$

is needed.

As usual, it is assumed that this problem has a unique solution $U = U(x,t)$, which has all the derivatives required during the presentation.

In the construction of locally one-dimensional scheme let formally replace the three-dimensional equation with one-dimensional chain of equations, i.e., approximate sequentially operators with $\frac{\tau}{3}$ increments:

$$\Re_\alpha U = \frac{1}{3}\frac{\partial U}{\partial t} - (L_\alpha U + f_\alpha), \alpha = 1,2,3, \qquad (9)$$

where f_α satisfies the condition in $\sum_{\alpha=1}^{3} f_\alpha = f$.

To approximate $L_\alpha U + f_\alpha$ in the spatial grid ω_h the homogeneous difference operator of second order approximation $\Lambda_\alpha y + \varphi_\alpha$ is used. The boundary conditions and the right side φ_α are taken at random moments of time:

$$\varphi_\alpha^{j+\frac{\alpha}{3}} = f_\alpha(x, t_{j+0.5}), \quad \mu^{j+\frac{\alpha}{3}} = \mu(x, t_{j+\alpha/3}), \quad \alpha = 1,2,3.$$

Approximating each heat conduction equation of number α in the interval $t_{j+(\alpha-1)/3} < t \le t_{j+\alpha/3}$ of two-layer scheme with weights, we obtain a chain p of one-dimensional schemes, called LOSs (local one-dimensional schemes):

$$\frac{U^{j+\frac{\alpha}{3}} - U^{j+\frac{\alpha-1}{3}}}{\tau} = \Lambda_\alpha\left(\sigma U^{j+\frac{\alpha}{3}} + (1-\sigma)U^{j+\frac{\alpha-1}{3}}\right) + \varphi_\alpha^{j+\frac{\alpha}{3}}, \quad \alpha = 1,2,3, \quad x \in \omega_h \qquad (10)$$

if σ - any number. If $\sigma = 1$, we get the number of implicit local one-dimensional scheme. If $\sigma = 0$, we get explicit LOC.

Let's look at implicit LOC:

$$\frac{y^{j+\frac{\alpha}{3}} - y^{j+\frac{\alpha-1}{3}}}{\tau} = \Lambda_\alpha y^{j+\frac{\alpha}{3}} + \varphi_\alpha^{j+\frac{\alpha}{3}}, \quad \alpha = 1,2,3, \quad x \in \omega_h \qquad (11)$$

Boundary condition:

$$y^{j+\frac{\alpha}{3}} = \mu^{j+\frac{\alpha}{3}}, \qquad (12)$$

$$x \in \gamma_{h,\alpha}, \quad j = 0,1,\dots,j_0, \ \alpha = 1,2,3$$

Initial condition:

(6`

$$y(x,0) = u_0(x) \qquad (13)$$

Let's look closely at (11):

$$\frac{y^{j+\frac{1}{3}} - y^j}{\tau} = \Lambda_1\left(y^{j+\frac{1}{3}}\right) + f_1^{j+\frac{1}{3}}$$

$$\frac{y^{j+\frac{2}{3}} - y^{j+\frac{1}{3}}}{\tau} = \Lambda_2\left(y^{j+\frac{2}{3}}\right) + f_2^{j+\frac{2}{3}}, \ j = \overline{1, m-1} \qquad (14)$$

$$\frac{y^{j+1} - y^{j+\frac{2}{3}}}{\tau} = \Lambda_3\left(y^{j+1}\right) + f_3^{j+1}$$

Boundary condition:

$$y^{j+\frac{\alpha}{3}}\Big|_{x_\alpha=0} = \mu\left(x, t_{j+\frac{\alpha}{3}}\right)\Big|_{x_\alpha=0}, \ y^{j+\frac{\alpha}{3}}\Big|_{x_\alpha=l_\alpha} = \mu\left(x, t_{j+\frac{\alpha}{3}}\right)\Big|_{x_\alpha=l_\alpha}, \ \alpha = 1,2,3 \qquad (15)$$

Initial condition:

$$y^0 = U_0(x) \qquad (16)$$

Let's examine each of three equations (14) separately. The first equation (14) is written as:

$$(y_1)_{i_1}^{j+1} - \tau\left(\frac{(k_1)_{i_1+1}^{j+1}\cdot(y_1)_{i_1+1}^{j+1} - \left((k_1)_{i_1+1}^{j+1} + (k_1)_{i_1}^{j+1}\right)\cdot(y_1)_{i_1}^{j+1} + (k_1)_{i_1}^{j+1}\cdot(y_1)_{i_1-1}^{j+1}}{h_1^2}\right) = \varphi_1^j, \quad (17)$$

$$j = \overline{0, m} - 1, i_1 = \overline{1, n_1 - 1}, i_2 = \overline{0, n_2}, i_3 = \overline{0, n_3},$$

$$f_1^j = f\left(x, t_j + \frac{\tau}{2}\right), j = \overline{1, m}$$

(17) rewrite as:

$$\left(1 + \frac{\tau}{h_1^2}\left((k_1)_{i_1+1}^{j+1} + (k_1)_{i_1}^{j+1}\right)\right)\cdot(y_1)_{i_1}^{j+1} - \frac{\tau}{h_1^2}(k_1)_{i_1+1}^{j+1}\cdot(y_1)_{i_1+1}^{j+1} - \frac{\tau}{h_1^2}(k_1)_{i_1}^{j+1}\cdot(y_1)_{i_1-1}^{j+1} = \varphi_1^j$$

Note as,

$$A_{i_1} = \frac{\tau}{h_1^2}(k_1)_{i_1}^{j+1}, C_{i_1} = 1 + \left(A_{i_1+1} + A_{i_1}\right), F_{i_1} = \varphi_1^j$$

we get:

$$A_{i_1}\cdot(y_1)_{i_1-1}^{j+1} - C_{i_1}\cdot(y_1)_{i_1}^{j+1} + A_{i_1+1}\cdot(y_1)_{i_1+1}^{j+1} = -F_{i_1},$$
$$i_1 = \overline{1, n_1 - 1}, i_2 = \overline{1, n_2 - 1}, i_3 = \overline{1, n_3 - 1} \quad (18)$$

Boundary conditions in general form:

$$(y_1)_0^{j+1} = \chi_1\cdot(y_1)_1^{j+1} + \mu_{11}^{j+1}, \quad (y_1)_{n_1}^{j+1} = \chi_2\cdot(y_1)_{n_1-1}^{j+1} + \mu_{12}^{j+1},$$

if $\chi_1 = 0, \chi_2 = 0$ - boundary conditions of 1st type, $\chi_1 = 1, \chi_2 = 1$ - boundary conditions of 2nd type.

Next, apply known value of y^0, we get F_1. Next, we solve the equation (18) for every grid note ω_h with sweep method:

$$\alpha_{i_1+1} = \frac{A_{i_1+1}}{C_{i_1} - A_{i_1}\cdot\alpha_{i_1}}, \alpha_1 = \chi_1,$$

$$\beta_{i_1+1} = \frac{A_{i_1+1}\cdot\beta_{i_1} + F_{i_1}}{C_{i_1} - A_{i_1}\cdot\alpha_{i_1}}, \beta_1 = \mu_{11}^0,$$

$$(y_1)_{n_1}^{j+1} = \frac{\mu_{12}^{j+1} + \chi_2\cdot\beta_{n_1}}{1 - \chi_2\cdot\alpha_{n_1}} \quad (21)$$

Solve other 2 equations (14) the same way. The solution is $(y_3)_{i_3}^{j+1} = \alpha_{i_1+1}\cdot(y_1)_{i_1+1}^{j+1} + \beta_{i_1+1}, i_1 = \overline{n_1 - 1, 0}, i_2 = \overline{1, n_2 - 1}, i_3 = \overline{1, n_3 - 1}$

III.The results of numerical calculations

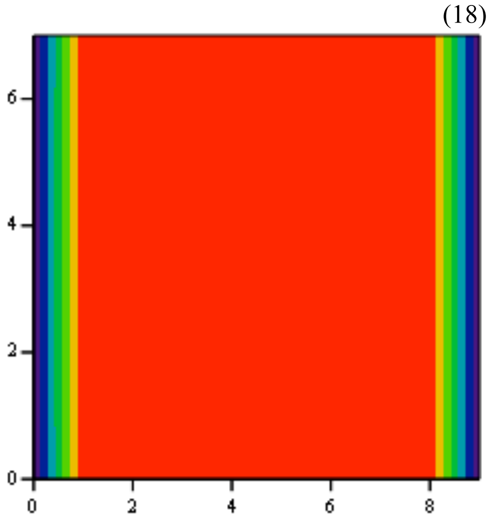
To test the proposed algorithm the problem of the temperature distribution of the strip of metal rod on the stands of continuous mill was numerically solved.

The developed algorithm for solving the problem of temperature modes considers the temperature changes directly in the rolling mill and in the interstand of the rolling mill. The algorithm can be used to calculate the temperature distribution of various rolled materials on any type of profiled rolling mill.

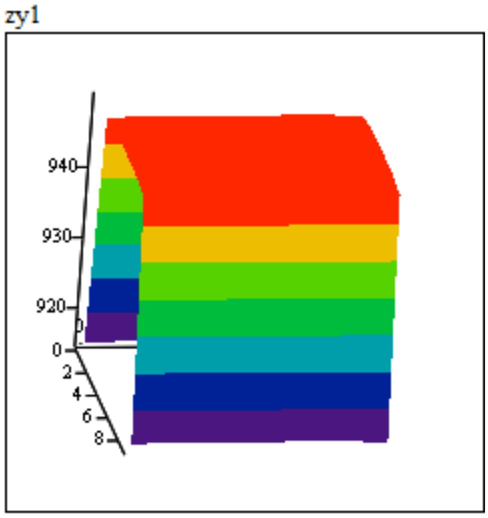
To implement the algorithm for determining the temperature regimes of bar rolling on a computer the program was written in the language Fortran PoverStation 4.0. For the calculation as input data the literature data were used.

As a result of the calculation the three-dimensional temperature field was obtained. The results of investigations of the temperature field of roll during the rolling metal alloy VT6 were shown. The preform object is heated to a temperature of 950 ^{0}C before the cage.

The results showed that the initially heated to a temperature of 950 ° C work piece put into the first roughing stand, after rolling in the stand the surface of the strip is cooled to a temperature in the range of 904-949 ° C. Figure 2 shows a cross-sectional view of roll after the first stand where the temperature of roll contacting with the roll, is reduced to 905 ° C due to heat transfer.



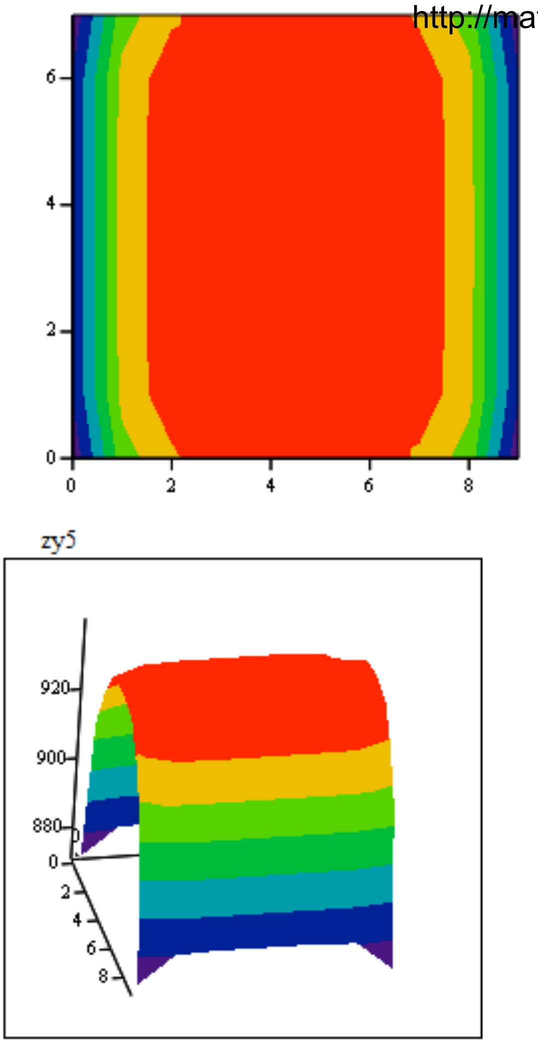
zy1



zy1

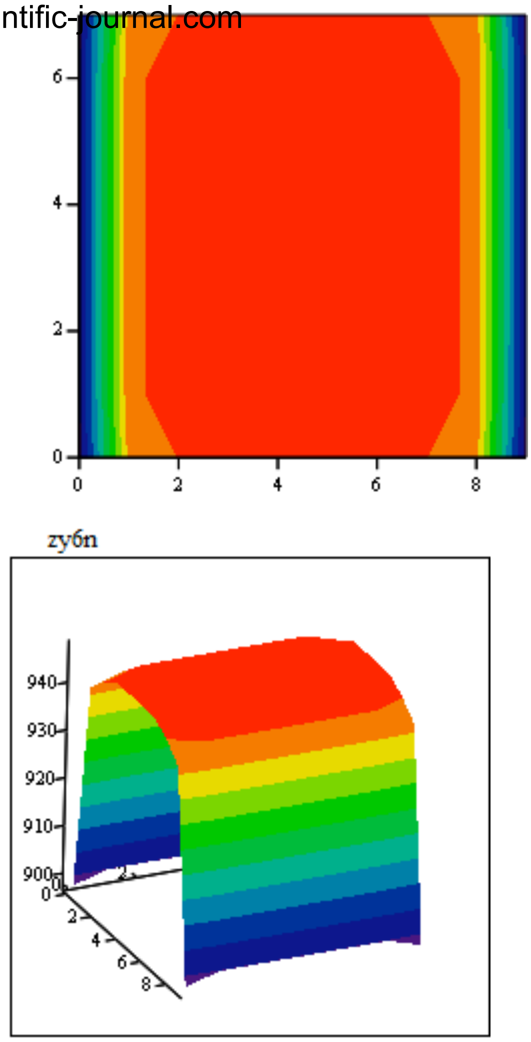
Fig. 2. The cross-section of roll after the first stand

Figure 3 shows a cross-sectional view after the roll of fifth stand, where the temperature of roll surface contacting with the roll, is reduced to 880 °C due to heat transfer, minimal drop in temperature - to 915 ° C is shown due to a greater plastic deformation of the work piece and greater angles deformation heating. Sides contacting only with the environment are cooled to a temperature of 938 ° C.



zy5

Fig. 3. The cross-section of roll cage after the fifth stand



zy6n

Fig. 4. The cross-section of roll cage after the sixth stand

After five passages in the stand, the strip is cut to length, cooled, and then reheated to a temperature of 950 ^{0}C. Further, it is rolled in a stand group of 6 passes of stands, i.e. 6 to 11 stands. The results are shown in Figures 4 and 5, respectively.

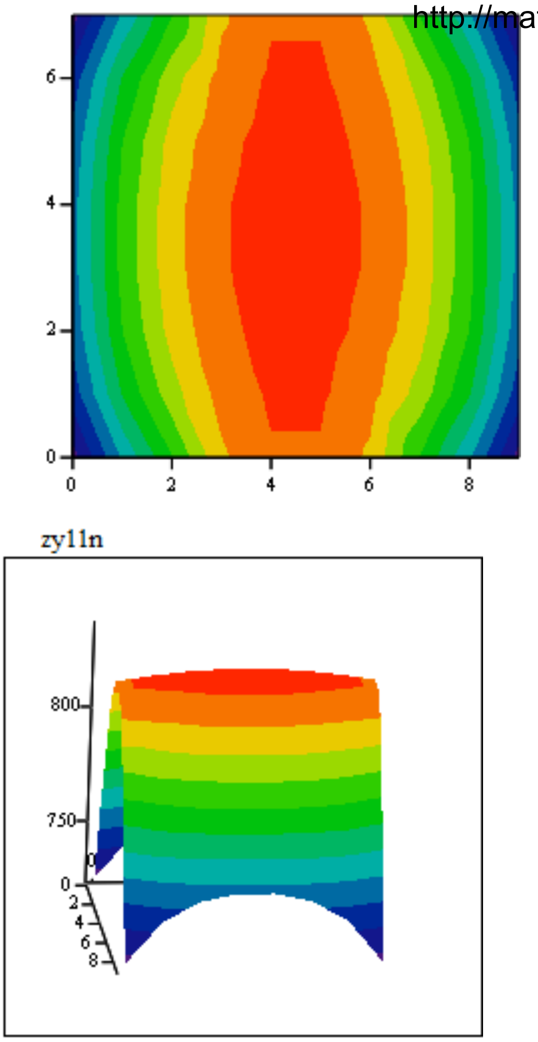


Fig. 5. The cross-section of roll cage after the 11th stand

To obtain the desired uniform structure is necessary to achieve a uniform temperature over the cross section of a metal rod. As seen in Figure 5, this condition is satisfied, although to carry out this condition is virtually impossible.

The resulting calculation by the implicit locally one-dimensional scheme of temperature of rolling over different parts of the passage is shown in Figure 6.

Studies show that the structure of the center and the surface of a metal rod are different due to the temperature differences about 50 ° C. The condition of heating the strip up to 950 ^{0}C at the entrance of the first and 6th stands are kept, i.e. temperature curve accurately describes the process of deformation of the metal rod.

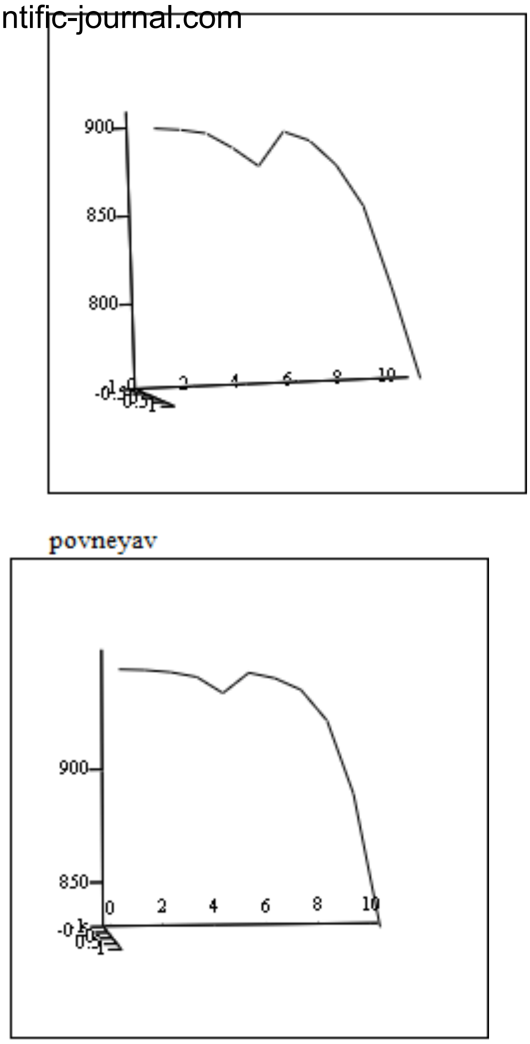


Fig. 6. The temperature difference curves according to the stands, calculated with implicit LOCs

(and, respectively, the temperature of the surface and central part of the roll)

IV. Conclusion

Determination of rolling temperature and temperature regimes is one of the most important technological factors, influencing the ductility and defect formation of deformable metal.

Checking the accuracy and comparative analysis of the calculation of temperature fields of rolling enables the use of the algorithm of locally one-dimensional schemes in the bar rolling.

Acknowledgment

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