Calculation of the temperature distribution field in the deformation zone in metal rod rolling with the aid of locally homogeneous scheme

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Abstract - On the basis of heat-conduction equation is solved the three-dimensional problem of temperature distribution field for continuous casting and rolling line of metal rod. Wherein considered the temperature rate change in rolling mill and intercellular gap of rolling mill line.

In order to solve three dimensional problem of heat-conduction is used the score approximation method of locally one-dimensional schemes. With the aid of which multivariate problem reduced to locally one-dimensional problem. Calculation of temperature field distribution in deformation zone is executed under inherent scheme.

Key words: continuous casting, a metal rod, distribution field, deformation zone, stand, locally one-dimensional scheme.

I. Introduction

During metal forming under pressure one of the main factors, along with the speed, the deformation and stress that affect the quality of the products, is the temperature distribution field in the deformation zone. However, the experimental determination of the distribution of the temperature field in an industrial environment is almost impossible. Therefore, for temperature control in the manufacturing of quality metal products the algorithm of solving three-dimensional problems of the temperature distribution field for the continuous casting line and metal rod rolling with the aid of continuous casting and metal rod rolling, one of the most widely used method is the method of nets.

Simplicity and flexibility are characterized for the difference methods of solving boundary value problems of mathematical physics in the regular settlement areas, but the use of irregular nets brings together the finite difference method with the finite element method.

One of the most important advances in computational mathematics is the cost difference methods for solving multidimensional equations (with several space variables $x_1,x_2,...,x_n$) in partial derivatives [3]. One of the economical schemes class is additive schemes, which have total approximation, which is unconditionally stable and requires the cost of arithmetic operations $Q$, proportional to the grid nodes $\Omega_{h}$, in order to move from one layer to another.

The method of total approximation provides a completely stable converging locally one-dimensional scheme for parabolic equations.

Developed algorithm based on locally one-dimensional scheme, that implements a mathematical model (1), allows us to solve the problem of controlling the temperature conditions of rolling a metal rod on a rolling mill stand.

The problem is solved by a difference method, locally one-dimensional schemes [3] on the basis of the differential equation of heat conduction [2] in the form of:

$$c\rho \frac{\partial U}{\partial t} = \text{div}(\lambda \cdot \text{grad} U) + \tau_s H,$$

if $c$ - thermal capacitance of metal; $\rho$ - mass density of metal; $\lambda$ - heat conduction coefficient; $\tau_s$ - metal resistance to plastic deformation of shift; $H$ - intensity of shift deformation rate, with following boundary conditions:

1. At the entrance in the deformation zone the temperature of the metal is known;

2. On the free surface there is the heat transfer by radiation to the environment, which is described by the Stefan-Boltzmann formula;

3. On the contact surface there is the heat exchange between the surfaces of the deformable metal and the rolls, which is described by the expression:

$$\lambda \frac{\partial U}{\partial n} = \alpha_s (U - U_b),$$

if $\alpha_s$ - heat transfer coefficient on the contact surface; $U_b$ - the temperature of rolls.
4. On other surfaces, bounding the deformation zone, the heat flow is zero.

In theory, in the most metal forming processes, the initial shape is different from the shape of the finished product, determined by the shape of the tool. The closer the element to the section angle, the less it receives the elongation. Therefore, section side receive convex shape. Square section will approach to the circular, and rectangular – any number. If \( h, x \) are taken at random, we get the number of point of 3-dimensional space with \( x \leq x \leq x, y \leq y \leq y, z \leq z \leq z \), the rolled profile, as well as rough gauges include simple shapes (rectangular, rhombus, oval, circle, square), the rectangular cross section was taken as the form of gauges of stands to experiment (Figure 1).

![Fig. 1. Rectangular calibration rolls of roughing mill.](image)

**II. Difference equation**

In order to construct the difference schemes for solving the heat conduction equation describing the temperature processes occurring in the deformation zone during the rolling of a metal rod, a three-dimensional parabolic equation of second order is consider [3]:

\[
\frac{\partial U}{\partial t} = \sum_{\alpha=1}^{3} L_\alpha U + f(x,t),
\]

where \( L_\alpha U = \frac{\partial}{\partial x_\alpha} \left( k(x,t) \frac{\partial U}{\partial x_\alpha} \right) \),

\( k(x,t) \geq c_i > 0, c_i = \text{const}, \)

if \( x = (x_1, x_2, x_3) \) - point of 3-dimensional space with \( x_1, x_2, x_3 \) coordinates. Let \( G \) – any 3-dimensional region with borders \( \Gamma, \mathcal{G} = G + \Gamma \),

\[
\overline{Q}_\tau = G \times [0 < t \leq T].
\]

The continuous solution of equation (6) in \( \overline{Q}_\tau \) cylinder, satisfying the boundary condition

\[
U = \mu(x,t) \text{ npth } x \in \Gamma, 0 \leq t \leq T,
\]

and initial condition

\[
U(x,0) = U_0(x), \text{ npth } x \in \mathcal{G}.
\]

is needed.

As usual, it is assumed that this problem has a unique solution \( U = U(x,t) \), which has all the derivatives required during the presentation.

In the construction of locally one-dimensional scheme let formally replace the three-dimensional equation with one-dimensional chain of equations, i.e., approximate sequentially operators with \( \frac{\tau}{3} \) increments:

\[
\mathcal{L}_\alpha U = \frac{1}{3} \frac{\partial U}{\partial t} - (L_\alpha U + f_\alpha), \alpha = 1, 2, 3,
\]

where \( f_\alpha \) satisfies the condition \( \sum_{\alpha=1}^{3} f_\alpha = f \).

To approximate \( L_\alpha U + f_\alpha \) in the spatial grid \( \omega_h \) the homogeneous difference operator of second order approximation \( \Lambda_\alpha y + \varphi_\alpha \) is used. The boundary conditions and the right side \( \varphi_\alpha \) are taken at random moments of time:

\[
\varphi_\alpha = f_\alpha(x,t_{j_{1/2}}), \quad \mu_\alpha = \mu(x,t_{j_{1/3}}), \quad \alpha = 1, 2, 3.
\]

Approximating each heat conduction equation of number \( \alpha \) in the interval \( t_{j_{1/2}} \leq t \leq t_{j_{1/3}} \) of two-layer scheme with weights, we obtain a chain \( \varphi \) of one-dimensional schemes, called LOSs (local one-dimensional schemes):

\[
\frac{U^{j+1} - U^j}{\tau} = \Lambda_\alpha \left( \alpha U^{j+1} + (1 - \alpha) U^j \right) + \varphi_\alpha, \quad \alpha = 1, 2, 3, \quad x \in \omega_h
\]

if \( \sigma \) - any number. If \( \sigma = 1 \), we get the number of implicit local one-dimensional scheme. If \( \sigma = 0 \), we get explicit LOC.

Let’s look at implicit LOC:

\[
\frac{y^{j+1/2} - y^j}{\tau} = \Lambda_\alpha \left( y^{j+1/2} + \varphi_\alpha \right), \quad \alpha = 1, 2, 3
\]

Boundary condition:

\[
\varphi = f_\alpha, \quad x \in y_{h, \alpha}, \quad j = 0, 1, ..., f_0, \quad \alpha = 1, 2, 3
\]

Initial condition:

\[
U^0(x) = u_0(x)
\]

Let’s look closely at (11):

\[
\frac{y^{j+1/2} - y^j}{\tau} = \Lambda_\alpha \left( y^{j+1/2} + f_\alpha \right) + f_1
\]

Boundary condition:

\[
\varphi = f_\alpha, \quad x \in y_{h, \alpha}, \quad j = 1, m - 1
\]

Initial condition:

\[
U^0(x) = u_0(x)
\]

Let’s examine each of three equations (14) separately. The first equation (14) is written as:
\[(y_{i_k})_{i-1} \left( \frac{h_i}{(k_i)^{i+1} - ((y_i)_{i+1} - (y_i)_{i+1}^{i})} + (k_i)_{i+1}^{i} - (y_i)_{i+1}^{i} \right) = q_{i_k}, \]

\[j = 0, m - 1, n_i = 1, n_2 - 1, i_2 = 0, n_3 - 1, i_3 = 0, n_3, \]

\[f_i^j = f(x_i, t_j + \frac{r}{2}), j = 1, m \]

(17) rewrite as:

\[\left(1 + \frac{r}{h_i}((k_i)_{i+1}^{i} + (k_i)_{i+1}^{i}) \right) (y_{i+1})_{i+1}^{i} - \frac{r}{h_i}((k_i)_{i+1}^{i} - (y_i)_{i+1}^{i} - \frac{r}{h_i}((k_i)_{i+1}^{i} - (y_i)_{i+1}^{i} = q_{i+1}^j. \]

Next, we apply known value of \( y_i^0 \), we get:

\[A_h \cdot (y_i)_{i+1}^{i+1} - C_h \cdot (y_i)_{i+1}^{i+1} + A_{i+1} \cdot (y_i)_{i+1}^{i+1} = -F_{i+1}, \]

\[i = 1, n_1 - 1, i_2 = 1, n_2 - 1, i_3 = 1, n_3 - 1. \]

Boundary conditions in general form:

\[(y_1)_{x_1} = \chi_1 \cdot (y_1)_{x_1} + \mu_1^{x_1}, (y_1)_{x_1} = \chi_2 \cdot (y_1)_{x_1} + \mu_2^{x_1}, \]

if \( \chi_1 = 0, \chi_2 = 0 \) - boundary conditions of 1st type, \( \chi_1 = 1, \chi_2 = 1 \) - boundary conditions of 2nd type.

Next, apply known value of \( y_i^0 \), we solve \( F_i \). Next, we solve the equation (18) for every grid note \( \omega_h \) with sweep method:

\[\alpha_{i+1} = \frac{A_h}{C_h - A_h \cdot \alpha_{i}}, \alpha_{i} = \chi_i, \]

\[\beta_{i+1} = \frac{A_h}{C_h - A_h \cdot \beta_{i}}, \beta_{i} = \mu_i, \]

Take other 2 equations (14) the same way. The solution is:

\[(y_1)_{i+1}^{i+1} = \chi_1 \cdot (y_1)_{i+1}^{i+1} + \mu_1^{i+1}, \]

\[i = 1, n_1 - 1, i_2 = 1, n_2 - 1, i_3 = 1, n_3 - 1. \]

III. The results of numerical calculations

To test the proposed algorithm the problem of the temperature distribution of the strip of metal rod on the stands of continuous mill was numerically solved.

The developed algorithm for solving the problem of temperature modes considers the temperature changes directly in the rolling mill and in the interstand of the rolling mill. The algorithm can be used to calculate the temperature distribution of various rolled materials on any type of profiled rolling mill.

To implement the algorithm for determining the temperature regimes of bar rolling on a computer the program was written in the language Fortran PowerStation.
Fig. 3. The cross-section of roll cage after the fifth stand

After five passages in the stand, the strip is cut to length, cooled, and then reheated to a temperature of 950 °C. Further, it is rolled in a stand group of 6 passes of stands, i.e. 6 to 11 stands. The results are shown in Figures 4 and 5, respectively.

Fig. 4. The cross-section of roll cage after the sixth stand
Fig. 5. The cross-section of roll cage after the 11th stand

To obtain the desired uniform structure is necessary to achieve a uniform temperature over the cross section of a metal rod. As seen in Figure 5, this condition is satisfied, although to carry out this condition is virtually impossible.

The resulting calculation by the implicit locally one-dimensional scheme of temperature of rolling over different parts of the passage is shown in Figure 6.

Studies show that the structure of the center and the surface of a metal rod are different due to the temperature differences about 50 °C. The condition of heating the strip up to 950 °C at the entrance of the first and 6th stands are kept, i.e. temperature curve accurately describes the process of deformation of the metal rod.

Fig. 6. The temperature difference curves according to the stands, calculated with implicit LOCs (and, respectively, the temperature of the surface and central part of the roll)

IV. Conclusion

Determination of rolling temperature and temperature regimes is one of the most important technological factors, influencing the ductility and defect formation of deformable metal.

Checking the accuracy and comparative analysis of the calculation of temperature fields of rolling enables the use of the algorithm of locally one-dimensional schemes in the bar rolling.

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